Characterization of the Constant Sign of the Green's Functions related to Ordinary and Functional Differential Equations

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This talk is devoted to the study of the sign of the Green's function related to the general linear n^{th}-order equation

$$T_n[M]\,u(t) \equiv u^{(n)}(t) + a_1(t)\,u^{(n-1)}(t) + \cdots + a_{n-1}(t)\,u'(t) + (a_n(t) + M)\,u(t) = 0,\ t \in [a,b],$$

coupled to different types of two-point boundary conditions, which include, among others, the so-called $(k, n-k)$ boundary conditions

$$u(a) = u'(a) = \cdots = u^{(k-1)}(a) = u(b) = u'(b) = \cdots = u^{(n-k-1)}(b) = 0,\ 1 \le k \le n-2,$$

and, for the fourth order case, the simply supported boundary conditions

$$u(a) = u''(a) = u(b) = u''(b) = 0.$$

By assuming that operator $T_n[\bar{M}]$ is disconjugate on the interval $[a,b]$ for a given $\bar{M}$, we describe the interval of values on the real parameter M for which the sign of the Green's function remains fixed on $[a,b] \times [a,b]$.

The characterization is given by means of the spectral theory, in fact, the extremes of the interval of the real parameter M are given as the eigenvalues of related problems.

In order to illustrate the applicability of the obtained results, we calculate the exact parameter intervals of constant sign Green's functions for particular operators. We remark that our method avoids the necessity of calculating the expression of the Green's function.

Other kind of functional equations as, for instance, the equations with reflection on the argument, in which the term $u(-t)$ appears on the equation, are also studied.

References

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